Generalized Chaplygin Gas, Accelerated Expansion and Dark Energy-Matter Unification

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We consider the scenario emerging from the dynamics of a generalized d-brane in a $(d + 1, 1)$ spacetime. The equation of state describing this system is given in terms of the energy density, ρ, and pressure, p, by the relationship $p = -A/\rho^\alpha$, where A is a positive constant and $0 < \alpha \leq 1$. We discuss the conditions under which homogeneity arises and show that this equation of state describes the evolution of a universe evolving from a phase dominated by non-relativistic matter to a phase dominated by a cosmological constant via an intermediate period where the effective equation of state is given by $p = \alpha\rho$.

I. INTRODUCTION

There is mounting evidence that the Universe at present is dominated by a smooth component with negative pressure, the so-called dark energy, leading to accelerated expansion. While the most obvious candidate for such component is vacuum energy, a plausible alternative is dynamical vacuum energy [1,2], or quintessence. These models most often involve a single field [2–11] or, in some cases, two coupled fields [12–14]. However, these models usually face fine-tuning problems, notably the cosmic coincidence problem i.e. the question of explaining why the vacuum energy or scalar field dominate the Universe only recently. In its tracker version, quintessence models address this problem in that the evolution of the quintessence energy density is fairly independent of initial conditions; however, this seems to be achieved at the expense of fine-tuning the potential parameters so that the quintessence energy density changes behaviour around the epoch of matter-radiation equality so as to overtake the matter energy density at present, driving the Universe into accelerated expansion. Moreover, for quintessence models with shallow potentials, the quintessence field has to be nearly massless and one expects radiative corrections to destabilize the ratio between this mass and the other known scales of physics; on the other hand, the couplings of such a light field to ordinary matter give rise to long-range forces, which should have been detected in precision tests of gravity within the solar system, and time dependence of the constants of nature [15].

Recently, it has been suggested that the change of behavior of the missing energy density might be regulated by the change in the equation of state of the background fluid instead of the form of the potential, thereby avoiding the abovementioned fine-tuning problems. This is achieved via the introduction, within the framework of FRW cosmology, of an exotic background fluid, the Chaplygin gas, described by the equation of state

$$p = -\frac{A}{\rho^\alpha} \ , \tag{1}$$

with $\alpha = 1$ and A a positive constant. Inserting this equation of state into the relativistic energy conservation equation, leads to a density evolving as

$$\rho = \sqrt{A + \frac{B}{a^6}} \ , \tag{2}$$

where a is the scale factor of the Universe and B is an integration constant. This simple and elegant model smoothly interpolates between a dust dominated phase where, $\rho \simeq \sqrt{B}a^{-3}$, and a De Sitter phase where $p \simeq -\rho$, through an intermediate regime described by the equation of state for stiff matter, $p = \rho$ [16]. Interestingly, this setup admits a well established brane interpretation as Eq. (1), for $\alpha = 1$, is the equation of state associated with the parametrization invariant Nambu-Goto d-brane action in a $(d + 1, 1)$ spacetime. This action leads, in the light-cone parametrization, to the Galileo-invariant Chaplygin gas in a $(d, 1)$ spacetime and to the Poincaré-invariant Born-Infeld action in a $(d, 1)$ spacetime (see [17] and references therein for a thorough discussion). Moreover, the Chaplygin gas is the only gas known to admit a supersymmetric generalization [17].

It is clear that this model has a bearing on the observed accelerated expansion of the Universe [19] as it

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automatically leads to an asymptotic phase where the equation of state is dominated by a cosmological constant, $8\pi G\sqrt{A}$. Subsequently, it has been shown that this model admits, under conditions, an inhomogeneous generalization which can be regarded as a unification of dark matter and dark energy [18]. The unification idea has received much attention recently [20–23]. For instance, in Ref. [21] it is suggested that dark matter might consist of quintessence (cosmon) lumps, and in Ref. [22] it is shown that spintessence-like models are generally unstable to formation of Q-Balls which behave as pressureless matter.

In this work, we consider the case of a generic α constant in the range $0 < \alpha \le 1$ and show that it interpolates betwwen a universe dominated by dust and a De Sitter one via a phase described by a "soft" matter equation of state, $p = \alpha\rho$ ($\alpha \ne 1$). We show that the model can be easily accomodated in the standard structure formation scenarios and does not leave any undesirable signature on the Cosmic Microwave Background power spectrum. Furthermore, we show that the model corresponds to a generalized Nambu-Goto action which can be interpreted as a perturbed d-brane in a $(d+1,1)$ spacetime.

II. THE MODEL

Our starting point is the Lagrangian density for a massive complex scalar field, Φ,

$$\mathcal{L} = g^{\mu\nu}\Phi^*_{,\mu}\Phi_{,\nu} - V(|\Phi|^2) \quad , \tag{3}$$

which, as suggested in Ref. [18], can be expressed in terms of its masss, m, as $\Phi = (\frac{\phi}{\sqrt{2}}m)\exp(-im\theta)$. The Lagrangian density (3) can then be rewritten as

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\left(\phi^2\theta_{,\mu}\theta_{,\nu} + \frac{1}{m^2}\phi_{,\mu}\phi_{,\nu}\right) - V(\phi^2/2) \quad . \tag{4}$$

This sets the scale of the inhomogeneity since, assuming that spacetime variations of ϕ correspond to scales greater than m^{-1}, then

$$\phi_{,\mu} << m\phi \quad . \tag{5}$$

This is in contrast with the work of Ref. [16], where spatial homogeneity is assumed, and it is clearly a quite relevant contribution to generalize the use of the Chaplygin gas equation of state into the cosmological description. In this (Thomas-Fermi) approximation, the resulting Lagrangian density can now be written as

$$\mathcal{L}_{TF} = \frac{\phi^2}{2}g^{\mu\nu}\theta_{,\mu}\theta_{,\nu} - V(\phi^2/2) \quad . \tag{6}$$

The corresponding equations of motion are given by

$$g^{\mu\nu}\theta_{,\mu}\theta_{,\nu} = V'(\phi^2/2) \quad , \tag{7}$$

$$(\phi^2\sqrt{-g}g^{\mu\nu}\theta_{,\nu})_{,\mu} = 0 \quad , \tag{8}$$

where $V'(x) \equiv dV/dx$. The field θ can be regarded as a velocity field provided $V' > 0$, i.e.

$$U^\mu = \frac{g^{\mu\nu}\theta_{,\nu}}{\sqrt{V'}} \quad , \tag{9}$$

so that on the mass shell $U^\mu U_\mu = 1$. It then follows that the energy-momentum tensor built from the Lagrangian density Eq. (6) takes the form of a perfect fluid whose thermodynamic variables can be written as

$$\rho = \frac{\phi^2}{2}V' + V \quad , \tag{10}$$

$$p = \frac{\phi^2}{2}V' - V \quad . \tag{11}$$

Imposing the covariant conservation of the energy-momentum tensor for an homogeneous and isotropic spacetime

$$\dot{\rho} + 3H(p + \rho) = 0 \quad , \tag{12}$$

where $H = \dot{a}/a$ is the expansion rate of the Universe, we get, for the generalized Chaplygin gas equation of state, Eq. (1), a generalized version of Eq. (2)

$$\rho = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \quad . \tag{13}$$

¿From Eqs. (10) and (11), we obtain

$$d \ln\phi^2 = \frac{d(\rho - p)}{\rho + p} \quad , \tag{14}$$

which, together with Eq. (1), leads to a relationship between ϕ^2 and ρ:

$$\phi^2(\rho) = \rho^\alpha(\rho^{1+\alpha} - A)^{\frac{1-\alpha}{1+\alpha}} \quad . \tag{15}$$

Further algebraic manipulation, introducing Eqs. (10), (11) and (14) into the Lagrangian density (6), shows that it is possible to establish a brane connection to this setting, as the resulting Lagrangian density has the form of a *generalized* Born-Infeld theory:

$$\mathcal{L}_{GBI} = -A^{\frac{1}{1+\alpha}}\left[1 - (g^{\mu\nu}\theta_{,\mu}\theta_{,\nu})^{\frac{1+\alpha}{2\alpha}}\right]^{\frac{\alpha}{1+\alpha}} \quad , \tag{16}$$

which clearly reproduces the Born-Infeld Lagrangian density for $\alpha = 1$. This Lagrangian density can be regarded as a d-brane plus soft correcting terms; indeed, expanding the root in Eq. (16) around $\alpha = 1$, one obtains:

$$\left[1 - X^{\frac{1+\alpha}{2\alpha}}\right]^{\frac{\alpha}{1+\alpha}} = \sqrt{1-X}$$
$$+ \frac{X\log(X) + (1-X)\log(1-X)}{4\sqrt{1-X}}(1-\alpha)$$
$$+ \frac{E+F+G}{32(1-X)^{3/2}}(1-\alpha)^2 + \mathcal{O}((1-\alpha)^3) \quad , \tag{17}$$

where $X \equiv g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}$ and

$$E = X(X-2)\log^2(X) \quad , \qquad (18)$$

$$F = -2X(X-1)\log(X)[\log(1-X)-2] \quad , \qquad (19)$$

$$G = (X-1)^2[\log(1-X)-4]\log(1-X) \quad . \qquad (20)$$

The potential arising from this model can be written as

$$V = \frac{\rho^{1+\alpha}+A}{2\rho^\alpha} = \frac{1}{2}\left(\Psi^{2/\alpha}+\frac{A}{\Psi^2}\right) \quad , \qquad (21)$$

where $\Psi \equiv B^{-(1-\alpha/1+\alpha)}a^{3(1-\alpha)}\phi^2$, which reduces to the duality invariant, $\phi^2 \to A/\phi^2$, and scale-factor independent potential for the Chaplygin gas.

The effective equation of state in the intermediate regime between the dust dominated phase and the De Sitter phase can be obtained expanding Eq. (13) in subleading order:

$$\rho \simeq A^{\frac{1}{1+\alpha}}+\left(\frac{1}{1+\alpha}\right)\frac{B}{A^{\frac{\alpha}{1+\alpha}}}a^{-3(1+\alpha)} \quad , \qquad (22)$$

$$p \simeq -A^{\frac{1}{1+\alpha}}+\left(\frac{\alpha}{1+\alpha}\right)\frac{B}{A^{\frac{\alpha}{1+\alpha}}}a^{-3(1+\alpha)} \quad , \qquad (23)$$

which corresponds to a mixture of vacuum energy density $A^{\frac{1}{1+\alpha}}$ and matter described by the "soft" equation of state:

$$p = \alpha\rho \quad . \qquad (24)$$

In broad terms, the comparison between the cosmological setting we propose and the one emerging from the Chaplygin gas, discussed in Refs. [16,18], is exhibited in Figure 1. Naturally, a complete cosmological scenario involves the inclusion of radiation, which is related to the massless degrees of freedom of the Standard Model at a given temperature and that were dominant before recombination. These clearly do not affect any of the features of the scenario we propose here. Less trivial, however, is the treatment of the inhomogeneities we have allowed in our setting. We analyse this issue in what follows.

Our starting point is Eq. (8), which can be shown to admit as first integral a position dependent function $B(\vec{r})$, after a convenient choice of comoving coordinates where the velocity field is given by $U^\mu = \delta_0^\mu/\sqrt{g_{00}}$ [18]. Taking for the metric $g_{\mu\nu}$, the proper time $d\tau = \sqrt{g_{00}}dx^0$, and $\gamma \equiv -g/g_{00}$ as the determinant of the induced 3-metric, then

$$\gamma_{ij} = \frac{g_{i0}g_{j0}}{g_{00}}-g_{ij} \quad . \qquad (25)$$

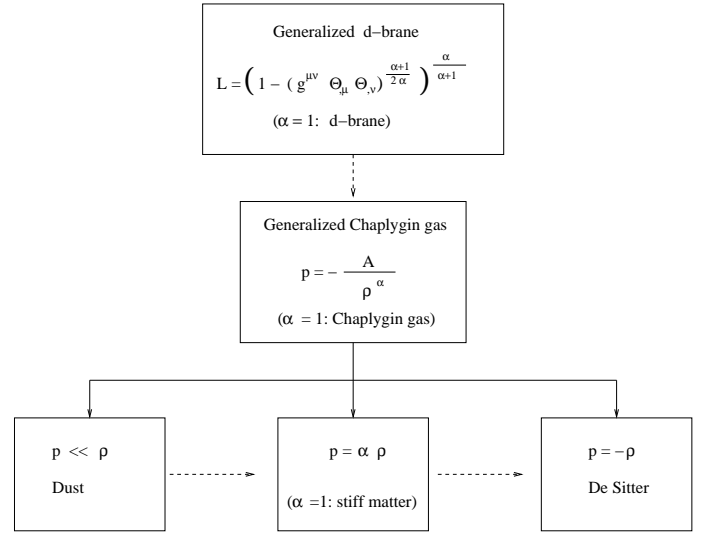Since for the relevant scales, function $B(\vec{r})$ can be regarded as approximately constant, we get

FIG. 1. Cosmological evolution of a universe described by a generalized Chaplygin gas equation of state.

$$\rho = \left(A+\frac{B}{\gamma^{(1+\alpha)/2}}\right)^{\frac{1}{1+\alpha}} \quad . \qquad (26)$$

This result suggests that the Zeldovich method for considering inhomogeneities can be implemented through the deformation tensor [18,24,25]:

$$D_i^j = a(t)\left(\delta_i^j-b(t)\frac{\partial^2\varphi(\vec{q})}{\partial q^i\partial q^j}\right) \quad , \qquad (27)$$

where $\vec{q}$ are generalized Lagrangian coordinates so that

$$\gamma_{ij} = \delta_{mn}D_i^m D_j^n \quad , \qquad (28)$$

and h is a perturbation

$$h = 2b(t)\varphi_{,i}{}^i \quad , \qquad (29)$$

with $b(t)$ parametrizing the time evolution of the inhomogeneities. Hence, using Eqs. above and Eqs. (22) - (23), it follows that

$$\rho \simeq \bar{\rho}(1+\delta) \quad , \quad p \simeq -\frac{A}{\bar{\rho}^\alpha}(1-\alpha\delta) \quad , \qquad (30)$$

where $\bar{\rho}$ is given by Eq. (13) and the density contrast, δ, is related to h through

$$\delta = \frac{h}{2}(1+w) \quad , \qquad (31)$$

and w reads

$$w \equiv \frac{p}{\rho} = -\frac{A}{\bar{\rho}^{1+\alpha}} \quad . \qquad (32)$$

The metric (28) leads to the following $0-0$ component of the Einstein equations:
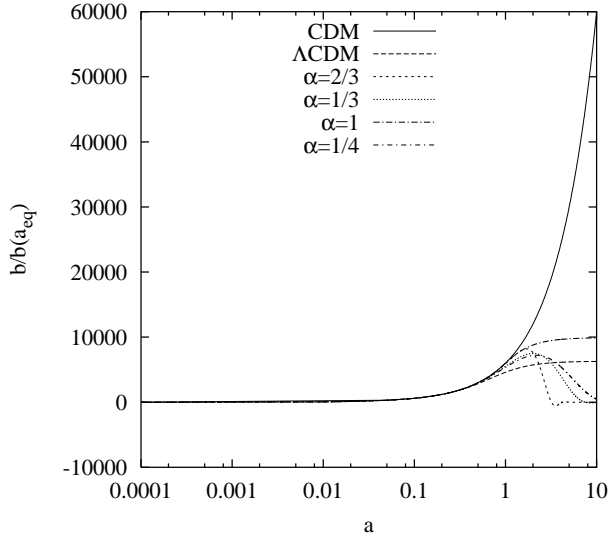
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FIG. 2. Evolution of $b(a)/b(a_{eq})$ for the generalized Chaplygin gas model, for different values of α, as compared with CDM and ΛCDM.

FIG. 3. Evolution of $b(a)/b(a_{eq})$ for the generalized Chaplygin gas model, for different values of α, as compared with ΛCDM.

$$-3\frac{\ddot{a}}{a} + \frac{1}{2}\ddot{h} + H\dot{h} = 4\pi G\bar{\rho}[(1+3w) + (1-3\alpha w)\delta] \quad , \tag{33}$$

where the unperturbed part of this equation corresponds to the Raychaudhuri equation

$$-3\frac{\ddot{a}}{a} = 4\pi G\bar{\rho}(1+3w) \quad . \tag{34}$$

Using the Friedmann equation for a flat spacetime

$$H^2 = \frac{8\pi G}{3}\bar{\rho} \quad , \tag{35}$$

Eq. (33) can be written as a differential equation for $b(a)$:

$$\frac{2}{3}a^2 b'' + (1-w)ab' - (1+w)(1-3\alpha w)b = 0 \quad , \tag{36}$$

where the primes denote derivatives with respect to the scale-factor, a.

Finally, from Eqs. (13) and (32), we derive an expression for w as a function of the scale-factor, using the observational input that $\Omega_\Lambda + \Omega_M = 1$, where $\Omega_\Lambda \simeq 2/3$ and $\Omega_M \simeq 1/3$ [26] are, respectively, the fractional vacuum and matter (dark + baryons) energy densities

$$w(a) = -\frac{\Omega_\Lambda a^{3(1+\alpha)}}{1 - \Omega_\Lambda + \Omega_\Lambda a^{3(1+\alpha)}} \quad . \tag{37}$$

We have used this expression to integrate Eq. (36) numerically, for different values of α. We have set $a_{eq} = 10^{-4}$ for matter-radiation equilibrium and $a_0 = 1$ at present, taking as initial condition $b'(a_{eq}) = 0$. Our results are shown in Figures 2 and 3.

We find that generalized Chaplygin scenarios start differing from the ΛCDM only recently ($z \simeq 1$) and that,
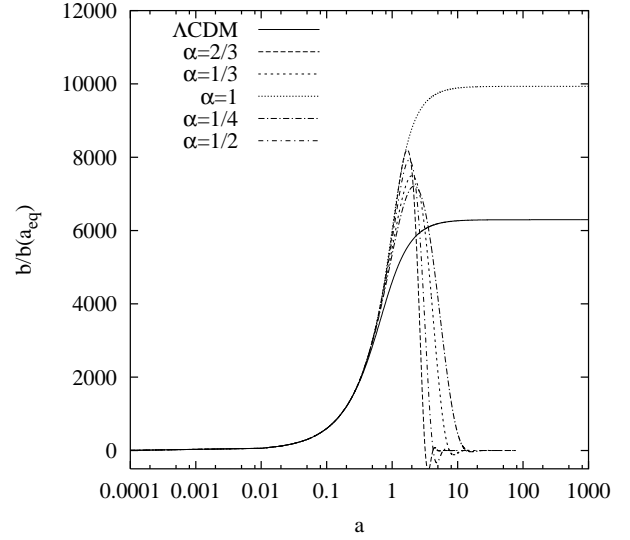
in any case, they yield a density contrast that closely resembles, for any value of $\alpha \neq 0$, the standard CDM before the present. Notice that the ΛCDM corresponds effectively to setting $\alpha = 0$ in Eq. (37) and removing the factor $1 - 3\alpha w$ in Eq. (36). Figure 3 shows also that, for any value of α, $b(a)$ saturates as in the ΛCDM case.

As for the density contrast, δ, we can see, using Eqs. (29), (31) and (37), that the ratio between this quantity in the Chaplygin and the ΛCDM scenarios is given by:

$$\frac{\delta_{Chap}}{\delta_{\Lambda CDM}} = \frac{b_{Chap}}{b_{\Lambda CDM}} \frac{1 - \Omega_\Lambda + \Omega_\Lambda a^3}{1 - \Omega_\Lambda + \Omega_\Lambda a^{3(1+\alpha)}} \quad , \tag{38}$$

meaning that their difference diminishes as a evolves. In Figure 4, we have plotted δ as a function of a for different values of α; hence, we verify for any α the claim of Refs. [18,27], for $\alpha = 1$, that the density contrast decays for large a. Figure 4 also shows the main difference in behaviour of the density contrast between a Universe filled with matter with a "soft" or "stiff" equations of state as the former resembles more closely the ΛCDM.

III. DISCUSSION AND CONCLUSIONS

In this work, we have considered a generalization of the Chaplygin equation of state, $p = -\frac{A}{\rho^\alpha}$, with $0 < \alpha \leq 1$. We have shown that, as in the case of the Chaplygin gas, where $\alpha = 1$, the model admits a d-brane connection as its Lagrangian density corresponds to the Born-Infeld action plus some soft logarithmic corrections. Furthermore, spacetime is shown to evolve from a phase that is initially dominated, in the absence of other degrees of freedom on the brane, by non-relativistic matter to a phase that is asymptotically De Sitter. This behaviour
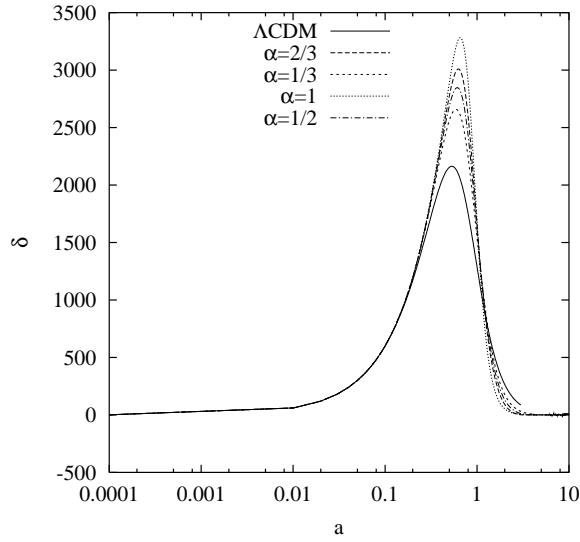
FIG. 4. Density contrast for different values of α, as compared with ΛCDM.

is similar to one of the Chaplygin gas. The intermediate regime in our model corresponds to a phase where the effective equation of state is given by $p = \alpha\rho$. We have estimated the fate of the inhomogeneities admitted in the model and shown that these evolve consistently with the observations as the density contrast they introduce is smaller than the one typical of CDM scenarios and closer to the ones predicted by the ΛCDM in comparison to the Chaplygin $\alpha = 1$ case.

Hence, given the fundamental nature of the underlying physics behind the Chaplygin gas and its generalizations, it appears that it contains some of the key ingredients in the description of the Universe dynamics at early as well as late times.

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